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TRANSLATION

WING FLUTTER AT NONLINEAR AERODYNAMIC FORCES

By

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Wing Flutter at Nonlinear Aerodynamic Forces

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Vibrations of an aircraft wing during flight should be considered as vibrations in a complex heterogeneous vibrational system with distributed parameters. But quite often to reveal the nature of the flutter phenomenon, or autoexcitation of elastic vibrations of a wing in flight, are used simplified systems [1,2]. One of such single arrangements is replacement of the wing by a thin rigid plate, elastically fastened so, that it can execute combined "bending-torsional" vibrations, consisting of progressive vertical vibrations of the plate as a single integral and torsional vibrations of same around a certain axis. At a proper selection of parameters such an arrangement does quite well represent vibrations of the right wing tip of no small aspect ratio. Analysis of phenomena on such a simplified system allows to examine the basic laws governing flutter and to obtain a dependence of the critical velocities upon the main dynamic parameters of the wing: natural frequencies of the wing and the magnitude of "bond betwen bending and buckling".

In a majority of existing experiments the flutter phenomenon is investigated in linear approximation, which is valid at small vibrations and at not high flight speeds. If the speed of flight is many times greater than the speed of sound, then in conformity with the theory, [1,3] the aerodynamic forces depend nonlinearly upon the angle of attack (Newton law) and this nonlinear dependence has to be taken into consideration.

At very high supersonic speeds the time $\tau = b/v$, where b - wing chord, v-flight speed, will be insignificantly small in comparison with the period of oscillations T. The ratio τ/T is proportional to the Strukhali number and in the case under consideration very close to zero, and consequently it can be assumed, that it is permissible

to employ the results of stationary theory, obtained for a constant in time angle of attack, for the case of variable angle at vibrations. The basis of this assumption will be explained below.

Analysis of wing autoexcitation conditions with the aid of the above described simplified scheme offers the possibility of explaining all basic features of the phenomenon, connected with the nonlinear nature of lift/angle of attach dependence.

Analytical solution of the mentioned problem in view of the nonlinear nature of equations is difficult and possible only in approximation.

In this report are given results of studying bending-torsional flutter of a wing with "Newton" lift with the aid of an electronic modeling machine type MN-7.

The obtained dependence of critical flight speed, at which flutter appears, upon wing parameters and initial perturbations, is described.. In spite of the fact that all measurements pertain only to qualitative description of phenomena, nonetheless it can be assumed that the possible value of critical rate of flutter can be determined theoretically approximately with the same accuracy, as it is done for small velocities in the case of applying the Bubnov-Galerkin method [1].

1. Simplified arrangement for studying bending-torsional vibrations of cantilever secured wing is shown in fig.1. Equations of vibrations of such a system have the form of

$$m\ddot{h} + kh - m\sigma b\ddot{\vartheta} = F, \quad I\ddot{\vartheta} + c\vartheta - m\sigma b\ddot{h} = M \qquad (1.1)$$

where h - displacement of axis of rotation (rigidity axis), ϑ- angle of rotation of wing around the rigidity axis, m-mass of wing, k-elasticity coefficient of wing at vertical displacements (bending rigidity), c-elasticity coefficient at rotation around axis (torsional rigidity of wing), I-moment of inertia relative to rigidity axis, b-wing chord, σb -distance from rigidity axis to the center of gravity, F and M aerodynamic forces and moment of these forces relative to the rigidity axis. In equations (1.1) the coordinates h and ϑ are read from the position of equilibrium in the absence of aerodynamic forces.

In the Newton approximation the resultant of aerodynamic forces in case of a time constant angle of attack was applied in the geometric center of the wing (if the wing is presented in form of a flat plate) and equals

$$F = Lb\rho v^2 \sin^2\theta \,\mathrm{sign}\,\theta \qquad \theta = \vartheta + \theta_0 \tag{1.2}$$

Here ϑ_0 - position of equilibrium in the absence of aerodynamic forces, L-length of wing, ρ-air density, v-speed of flight.

Moment of aerodynamic forces relative to the rigidity axis equals

$$M = Lb^2\sigma_0 \rho v^2 \sin^2\theta \,\mathrm{sign}\,\theta \tag{1.3}$$

where $\sigma_0 b$ - distance from rigidity axis to the point of applying the equivalent F.

At a variable angle of wing attack the magnitude of the aerodynamic force acting at the given point of the wing, is determined by instantaneous local angle of attack θ^*, which at plate vibrations will have at the distance zb from leading edge, the following value:

$$\theta^* = \theta + \frac{1}{v}\left[\dot\theta b\left(z + \sigma_0 - \frac{1}{2}\right) - \dot h\right] \tag{1.4}$$

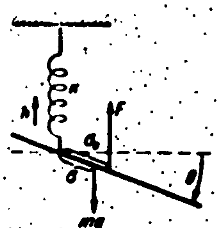

Fig. 1. Simplified arrangement of cantilever secured wing.

Consequently, the total lift affecting the wing during vibrations, equals

$$F = Lb\rho v^2 \int_0^1 \sin^2\theta^* \,\mathrm{sign}\,\theta^* dz \tag{1.5}$$

or replacing $\sin\theta^* dz$, we will obtain

$$F = Lb\rho v^2 \int_0^1 \theta^{*2} \,\mathrm{sign}\,\theta^* dz \tag{1.6}$$

In an analogous manner can also be presented the moment of aerodynamic forces M.

Substituting the value θ^* from (1.4) in (1.6), we have

$$F = Lb\rho v^2 \int_0^1 \left\{\theta + \frac{1}{v}\left[\dot\theta b\left(z + \sigma_0 - \frac{1}{2}\right) - \dot h\right]\right\}^2 \times$$
$$\times \,\mathrm{sign}\left\{\theta + \frac{1}{v}\left[\dot\theta b\left(z + \sigma_0 - \frac{1}{2}\right) - \dot h\right]\right\} dz \tag{1.7}$$

and analogously

$$M = -Lb^2\rho v^2 \int_0^1 \left\{\theta + \frac{1}{v}\left[\dot\theta b\left(z + \sigma_0 - \frac{1}{2}\right) - \dot h\right]\right\}^2 \left(z + \sigma_0 - \frac{1}{2}\right) \times$$
$$\times \,\mathrm{sign}\left\{\theta + \frac{1}{v}\left[\dot\theta b\left(z + \sigma_0 - \frac{1}{2}\right) - \dot h\right]\right\} dz \tag{1.8}$$

It is apparent, that at small z the sign of the expression

$$\left\{\theta + \frac{1}{v}\left[\dot{\theta} b\left(z + \sigma_0 - \frac{1}{2}\right) - \dot{h}\right]\right\} \quad (1.9)$$

is determined by the sign of the value

$$\left\{\theta + \frac{1}{v}\left[\dot{\theta} b\left(\sigma_0 - \frac{1}{2}\right) - \dot{h}\right]\right\} \quad 1.9a$$

Expression (1.9) changes sign at

$$z = z_0 = \left[\frac{v}{b}\frac{\theta}{\dot{\theta}} + \sigma_0 - \frac{1}{2} - \frac{\dot{h}}{b\dot{\theta}}\right] \quad 1.9b$$

If z_0 is beyond the limits of integration, then this change of sign needs not be considered. Consequently to calculate integrals in (1.7(and (1.8) we shall examine two cases.

1. Assuming that $z_0 > 1$, then, as it can be easily seen,

$$F = Lb\rho v^2\left(\theta^2 + 2S\sigma_0\theta\theta' - 2S\theta x'\right)\operatorname{sign}\left[\theta + S\theta'\left(\sigma_0 - \frac{1}{2}\right) - Sx'\right]$$

$$M = Lb^2\rho v^2\left[\theta^2 + 2S\left(\sigma_0 + \frac{1}{12\sigma_0}\right)\theta\theta' - 2S\theta x'\right] \times \quad (1.10)$$

$$\times \operatorname{sign}\left[\theta + S\theta'\left(\sigma_0 - \frac{1}{2}\right) - Sx'\right]$$

Here ω – frequency of bending vibrations of the wing, S-value, proportional to Strukhali number, x – dimensionless coordinate of vertical displacement of the wing, dashes designate differentiation in accordance with dimensionless time τ

$$x = \frac{h}{b}, \quad \tau = \omega t, \quad S = \frac{\omega b}{v}, \quad \omega = \sqrt{\frac{k}{m}} \quad 1.10a$$

At small values of the Strukhali number, when $S \ll 1$, the condition $z_0 \geq 1$ is fulfilled everywheres. In addition, in expressions (1.10) and (1.11) it is possible to discard all members, containing derivatives, because they will be small in [illegible]ison with θ and θ^2. We then obtain expressions, coinciding with with (2) and (3).

2. At $z_0 < 1$. In this case the zone of integration in (1.7) and (1.8) should be divided into two zones: $0 \leq$ [illegible] z_0 and $z_0 \leq z \leq 1$. We will then obtain

$$F = Lb\rho v^2\Big\{\left[\theta^2 + 2S\sigma_0\theta\theta' - 2S\theta x' \right] + [illegible]$$

$$+ \left(\frac{\theta}{S\theta'} + \sigma_0 - \frac{1}{2} - \frac{x'}{\theta'}\right)\left(\theta^2 + S\left(z_0 - 1 + 2\sigma_0\right)\theta\theta' - 2S\theta x'\right)\Big]\operatorname{sign}\theta' -$$

$$- \left(\frac{\theta}{S\theta'} + \sigma_0 - \frac{1}{2} - \frac{x'}{\theta'}\right)\left[\theta^2 + S\left(z_0 - 1 + 2\sigma_0\right)\theta\theta' - 2S\theta x'\right] \times$$

$$\times \operatorname{sign}\left[\theta + S\theta'\left(\sigma_0 - \frac{1}{2}\right) - Sx'\right]\Big\} \quad (1.11)$$

$$
\begin{aligned}
M = -Lb^2\rho v^2 \Big\{ \Big[& \theta^2\sigma_0 + 2S\sigma_0\theta\theta'\left(\sigma_0 + \frac{1}{12z_0}\right) - 2S\sigma_0\theta x' + \\
& + \left(\frac{\theta}{S\theta'} + \sigma_0 - \frac{1}{2} - \frac{x'}{\theta'}\right)\left(\theta^2\left(\frac{z_0}{2} - \frac{1}{2} + \sigma_0\right) - S\theta x'(z_0 - 1 + 2\sigma_0) + \right. \\
& \left. + 2S\theta\theta'\left(\frac{z_0^2}{3} + z_0\left(\sigma_0 - \frac{1}{2}\right) + \left(\sigma_0 - \frac{1}{2}\right)^2\right)\right)\Big] \operatorname{sign}\theta' - \\
& - \frac{1}{2}\left(\frac{\theta}{S\theta'} + \sigma_0 - \frac{1}{2} - \frac{x'}{\theta'}\right)\Big[\theta^2(z_0 - 1 + 2\sigma_0) + \\
& + 4S\theta\theta'\left(\frac{1}{3}z_0^2 + z_0\left(\sigma_0 - \frac{1}{2}\right) + \left(\sigma_0 - \frac{1}{2}\right)^2\right) - \\
& - 2S\theta x'(z_0 - 1 + 2\sigma_0)\Big] \operatorname{sign}\Big[\theta\Big] + Sv'\left(\sigma_0 - \frac{1}{2}\right) - Sx'\Big\}
\end{aligned}
\qquad (1.12)
$$

Expressions (1.11) and (1.12) are valid at relatively greater Strukhal' numbers.

In this report is discussed only the first approximation, corresponding to small Strukhali numbers. Solution of the problem in this case is much simpler, and for many practically important conditions such approximation if perfectly sufficient. It allows to explain the basic essential traits of the phenomenon and the influence of various parameters.

We will write equations (1) in a different form, solving same with respect to $\ddot{h}$ and $\ddot{\vartheta}$ and introducing new dimensionless values

$$\xi^2 = \frac{c}{I\omega^2}, \quad \chi = \frac{mb^2}{I}, \quad a^2 = \frac{1}{1 - \chi\sigma^2}, \quad q = \frac{L\omega^2}{k} \quad 1/12$$

Then we will obtain

$$\begin{aligned} x'' + a^2 x + a^2\xi^2\sigma\theta &= qa^2(1 - \chi\sigma_0\sigma)\sin^2\theta \operatorname{sign}\theta \\ \theta'' + a^2\xi^2\theta + a^2\chi\sigma x &= qa^2\chi(\sigma - \sigma_0)\sin^2\theta \operatorname{sign}\theta \\ \theta &= \vartheta - \theta_0 \end{aligned} \qquad (1.13)$$

2. Basic results. Equations (1.4) have been modeled on the MN-7 type machine. During the modeling in equation (1.13) have been introduced small members, determining oscillation damping. The magnitude of these small members, as shown by experiments, has practically no effect on the magnitude of the critical speed.

The examined system is nonlinear, consequently the conditions of excitation depend upon the initial state of the system. Measurement of the critical flight speed value, at flutter does take place, was carried out at initial conditions $x(0) = 0$ and $\vartheta(0) = 0$.

Such initial conditions indicate the presence of an initial deviation of the system from its stationary state x and ϑ, determinable by equation

$$\xi^2\theta^\circ = -\chi\sigma_0 q \sin^2\theta^\circ, \quad \vartheta = \theta^\circ + \theta_0, \quad x^\circ = q\sin^2\theta^\circ \qquad (2.1)$$

The mentioned initial perturbation may originate, for example, on account of the sharp change in airspeed as result of sudden wind gusts, atmospheric turbulence etc. Detailed investigations on the behavior of the system at various initial per

turbations have not been conducted in this experiment, and the problem was solved only at sufficiently high initial jolts for the purpose of explaining the effect of nonlinearity. For example, the case $x(0) = 0$; $\vartheta(0) = 0$ corresponds to the sudden rise in flight speed from zero to a value, corresponding to a given q, which, of course, is practically absolutely unrealizable. It is therefore only naturally to expect, that real values of critical speed will be higher than the ones obtained in this experiment (at corresponding wing parameters).

The case of small initial deviation from the equilibrium position of the system in the presence of aerodynamic forces, i.e. $x(0) \approx x^\circ$, $\vartheta(0) \approx \vartheta^\circ$ was not thoroughly investigated on the model, because in this case equations (1.13) can be linearized, by writing

$$\psi = \theta - \theta^\circ, \qquad y = x - x^\circ \tag{2.2}$$

where ϑ° and x° are determined from equations (2.1) which are easily solvable, by changing $\sin \theta^\circ$ into θ°

$$\theta^\circ = -\frac{\xi^2}{2\alpha\sigma_0 q} + \left(\frac{\xi^4}{4\alpha^2\sigma_0^2 q^2} + \frac{\xi^2\theta_0}{\alpha\sigma_0 q}\right)^{1/2}, \qquad x^\circ = q\theta^{\circ 2} \tag{2.3}$$

Substituting (2.2) in (1.13) and considering ψ and y as small values, we will obtain for them linear equations

$$\begin{aligned} y'' + a^2 y + a^2\xi^2\sigma\psi &= 2qa^2(1 - \alpha\sigma\sigma_0)\theta^\circ\psi \\ \psi'' + a^2\xi^2\psi + a^2\alpha\sigma y &= 2qa^2\alpha(\sigma - \sigma_0)\theta^\circ\psi \end{aligned} \tag{2.4}$$

A study of equations (2.4) for stability offers the following equation for critical value q_*

$$\alpha\sigma_0 \sin 2\theta_*^\circ q_* = \frac{\alpha^2}{4}\left[1 + \xi^2 - q_*\alpha(\sigma - \sigma_0)\sin 2\theta_*^\circ\right]^2 - \xi^2 \tag{2.5}$$

where θ_*° - determined by expression (2.3) at $q = q_*$

Table

ξ^2	$\sigma_0 = 0.25$		$\sigma_0 = 0$	
	Theory q_*	Model q_*	Theory q_*	Model q_*
2	2.2	2.0	0.7	0.55
4	8.0	6.6	1.6	1.3
52	13.0	10.8	2.1	1.75
6	-	-	2.7	2.3

Accurate calculation by formula (2.5) is quite cumbersome, but approximate evaluations can be obtained easily, since the value ϑ° depends weakly upon q. Results of calculating q_* at alpha = 7, σ = 0.25 for values $\sigma_0 = 0$ and $\sigma_1 = 0.25$ are listed in table. In this table for the purpose of comparing are given q_*m values obtained on the model and showing, that the model does quite well describe the oscillations in the system under question. The observed error bears a systematic nature and is explained, most likely, by inaccurate adjustment of the nonlinear block of the model issuing the function $\sin^2\vartheta$, and also by the approximation of the calculation.

At greater initial deviations of the system from the stationary state of equation (1.13) no linearization can be made, because nonlinearity here leads to essential differences in the behavior of the system as compared with linear theory.

Results of investigating in this case, obtained on model of various wing parameters, are given in fig. 2-6.

In fig. 2 is given the dependence of critical value of relative pressure q_* (flight speed is proportional to value $\sqrt{q_*}$) of torsional rigidity of the wing, characterized by value xi^2, for various positions of wing CG relative to the rigidity axis under conditions that the rigidity axis is placed in the geometrical center of the wing, i.e. $\sigma_0 = 0$. When the CG is displaced to the rear from the rigidity axis ($\sigma = 0.25$), in the system is observed a peculiar "resonance", i.e. critical speed decreases in this case, when the frequency of torsional oscillations is approximately doubly higher than the frequency of bending oscillations ($xi \approx 2$). This phenomenon is connected with the nonlinear quadratic dependence of the value of lift upon the angle of attack and appears only at consi-

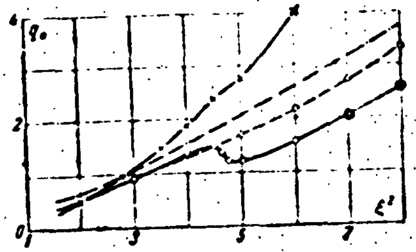

Fig. 2. Dependence of critical value q upon torsional rigidity of the wing at $\alpha = 7$, $\sigma_0 = 0$ for various CG positions: $\sigma = 0.1$ and $\sigma = 0.25$. Solid curves correspond to initial conditions $x(0)=0$, $\vartheta(0) = 0$. lower curve for $\sigma = 0.25$; dotted curve - $x(0) = x^{\circ}$, $\vartheta(0) = \vartheta^{\circ}$; dot-dash curve (theoretical) at $x(0) = x^{\circ}$, $\vartheta(0) = \vartheta^{\circ}$.

derable deviations of the initial position of the system from the stationary state. When the CG draws closer to the rigidity axis (e.g. σ = 0.1) "resonance" disappears and critical speed increases. When the CG is shifted in opposite direction of the rigidity axis flutter does not originate at all.

Fig.3 corresponds to the position of rigidity axis by a distance σ_0 = 0.25 from the center of the wing closer to its forward edge. When CG coincides with geometrical center of the wing ($\sigma = \sigma_0$ = 0.25) in the system is also observed "resonance". When the CG draws closer to the rigidity axis (σ = 0.2) the critical speed value rises sharply, and at $\sigma \approx 0.17$ flutter disappears. On the other hand, when CG is shifted in opposite direction (σ= 0.3) critical speed decreases and "resonance" disappears. The dependence of critical speed upon the position of CG in the case under consideration at constant xi^2 is given in fig.4.

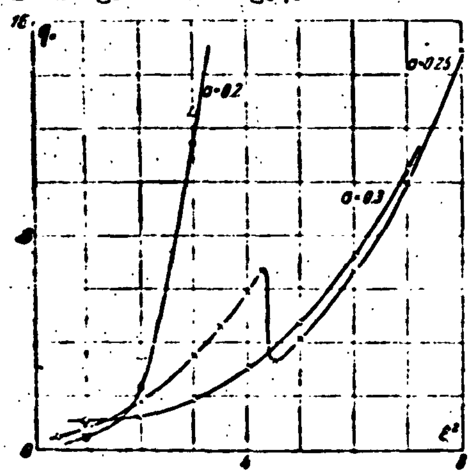

Fig.3. Dependence of critical value q_* upon the weight of the wing at torsion at alpha = 7, σ_0 =0.25 for various positions of CG: σ = 0.2; 0.25; 0.3

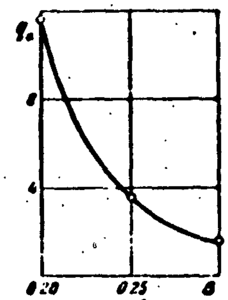

Fig.4. Dependence of critical value q_* upon the CG position σ at α = 7, σ_0 - 0.25, xi^2 = 3

A change in parameter a (change in magnitude of radius square of wing inertia) leads to displacement of "resonance" frequency, because at smaller a "resonance" occurs at greater torsional frequency ratios to the bending frequency (fig.5). Resonance becomes more pronounced here. At greater a values "resonance" disappear entirely (e.g. a = 10).

In (fig.6) is given the dependence q upon xi^2 in case when the rigidity axis is shifted by a distance σ_0 = 0.25 toward the trailing edge of the wing.

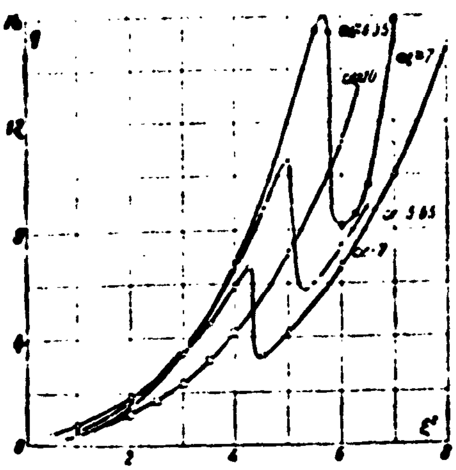

Fig.5. Dependence of critical value q_* of wing rigidity Ki^2 upon torsion at $\sigma_0 = 0.25$ and $\sigma = 0.25$ for various values of parameter a.

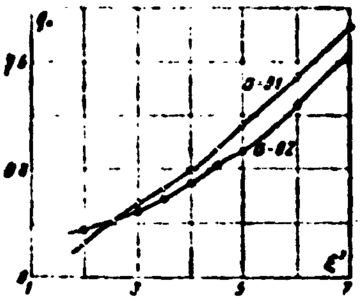

Fig.6. Dependence of critical value q_* of wing rigidity Ki^2 upon torsion in case $\sigma_0 = -0.25$, $\alpha = 7$; $\varepsilon = 0.1$ and $\sigma = 0.2$

In this case the critical pressure value rises almost smoothly and "resonance" is not observed.

In this experiment was also investigated the effect of fuselage on the magnitude of critical speed, at which flutter does take place. In this case was used a wing model, shown in fig.7.

Equations of oscillations of such a model have the form of

$$
\begin{aligned}
&m\ddot{h} + k(h - H) - m\sigma b\ddot{\theta} = F\\
&I\ddot{\theta} + c\theta - m\sigma b\ddot{h} = M\\
&M_0\ddot{H} + k(H - h) + k_0 H = 0
\end{aligned}
\qquad (2.6)
$$

where M_0-mass of fusealge, H-displacement of fuselage relative to the equilibrium position, k_0 - rigidity of suspension.

The investigation was made at $M_0/m = 10$, $k_0/k = 0.7$, $\sigma_0 = 0.25$. The results showed that in this case the consideration of the fuselage has a very slight effect on the magnitude of critical flutter rate.

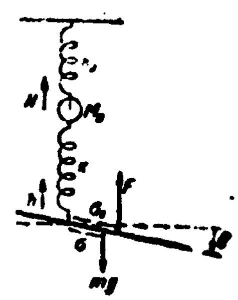

Fig.7. Simplified system of wing with fuselage.

The critical speed values with consideration of the fuselage are shown in graph in fig.3 by the sign Δ at $\sigma = 0.2$; a = 7. For other parameter values the change in critical speed with consideration of the fuselage was found to be even more insignificant.

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